1 The curve **C** has equation $y = 5x^3 - x^2 - 6x + 4$

(a) Find $\frac{dy}{dx}$

$\frac{d}{dx} ax^n = anx^{n-1}$

$\frac{dy}{dx} = 15x^2 - 2x - 6$

$\frac{dy}{dx} =$ $15x^2 - 2x - 6$ (2)

(2)

There are two points on the curve **C** at which the gradient of the curve is 2

(b) Find the x coordinate of each of these two points.
Show clear algebraic working.

when gradient = 2, $\frac{dy}{dx} = 2$

$15x^2 - 2x - 6 = 2$ (1)

$15x^2 - 2x - 8 = 0$ (1) $\quad -2$

Finding values of x :

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(15)(-8)}}{2(15)}$ (1)

$= \frac{2 \pm \sqrt{484}}{30}$

$= \frac{2 \pm 22}{30} \Rightarrow \frac{24}{30}, \frac{-20}{30}$

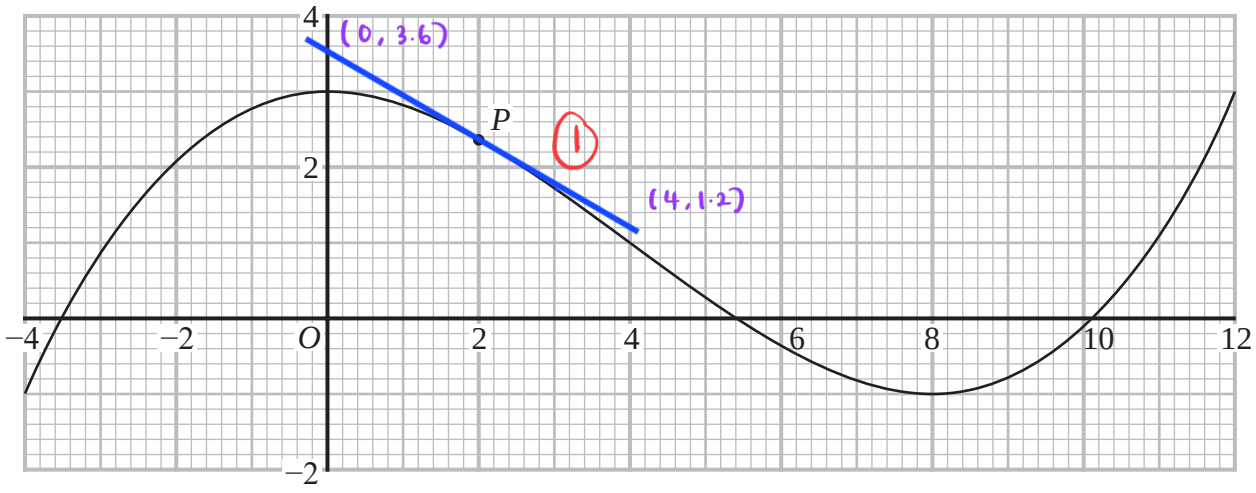
$= \frac{4}{5}, -\frac{2}{3}$ (1)

$x = \frac{4}{5}, -\frac{2}{3}$

(4)

(Total for Question 1 is 6 marks)

2 The diagram shows the graph of $y = f(x)$ for $-4 \leqslant x \leqslant 12$



The point P on the curve has x coordinate 2

(a) (i) Use the graph to find an estimate for the gradient of the curve at P.

$m = \dfrac{3.6 - 1.2}{0 - 4}$ (1)

$= -0.6$

-0.6 (1)

(3)

(ii) Hence find an equation of the tangent to the curve at P.
Give your answer in the form $y = mx + c$

$y = mx + c$ ← y-intercept

$y = -0.6x + 3.6$ (1)

y-intercept $= 3.6$

(1)
$y = -0.6x + 3.6$

(2)

The equation $f(x) = k$ has exactly two different solutions for $-4 \leqslant x \leqslant 12$

(b) Use the graph to find the two possible values of k.

-1 (1) , 3 (1)

(2)

(Total for Question 2 is 7 marks)

The curve **C** has equation $y = 2(x + 4)^2 - 12(x + 4) + 3$

The point M is the minimum point on **C**

3 (b) Find the coordinates of M

$y = 2x^2 + 16x + 32 - 12x + 48 + 3$

$= 2x^2 + 4x + 83$

$\frac{dy}{dx} = 4x + 4 = 0$

$x = \frac{-4}{4} = -1$ (1)

$y = 2(-1+4)^2 - 12(-1+4) + 3$

$= 2(3)^2 - 12(3) + 3$

$= 18 - 36 + 3$

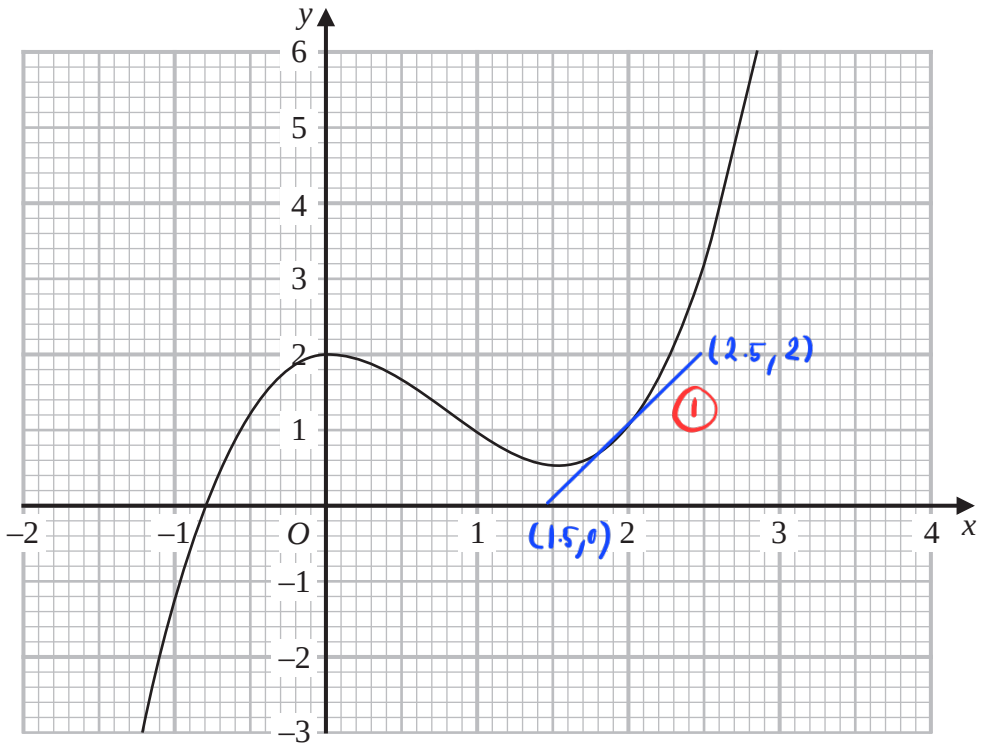
$= -15$ (1)

(-1 , -15)

(2)

(Total for Question 3 is 2 marks)

4 Part of the curve with equation $y = f(x)$ is shown on the grid.



Find an estimate for the gradient of the curve at the point where $x = 2$
Show your working clearly.

$m = \frac{(2-0)}{2.5-1.5} = 2$ (1)

(1)

2

(Total for Question 4 is 3 marks)

5 The curve **C** has equation $y = ax^3 + bx^2 - 12x + 6$ where a and b are constants.

The point A with coordinates $(2, -6)$ lies on **C**
The gradient of the curve at A is 16

Find the y coordinate of the point on the curve whose x coordinate is 3
Show clear algebraic working.

$-6 = a(2)^3 + b(2)^2 - 12(2) + 6$

$-6 = 8a + 4b - 24 + 6$

$8a + 4b = 12$ $-$ ① (1)

gradient, $\frac{dy}{dx} = 3ax^2 + 2bx - 12$ (1)

$16 = 3a(2)^2 + 2b(2) - 12$

$16 = 12a + 4b - 12$

$4b = 28 - 12a$ $-$ ② (1)

② into ① :

$8a + 2(8 - 12a) = 12$

$-4a = -16$

$a = 4$

$b = -5$ (1)

$y = 4(3)^3 - 5(3)^2 - 12(3) + 6$ (1)

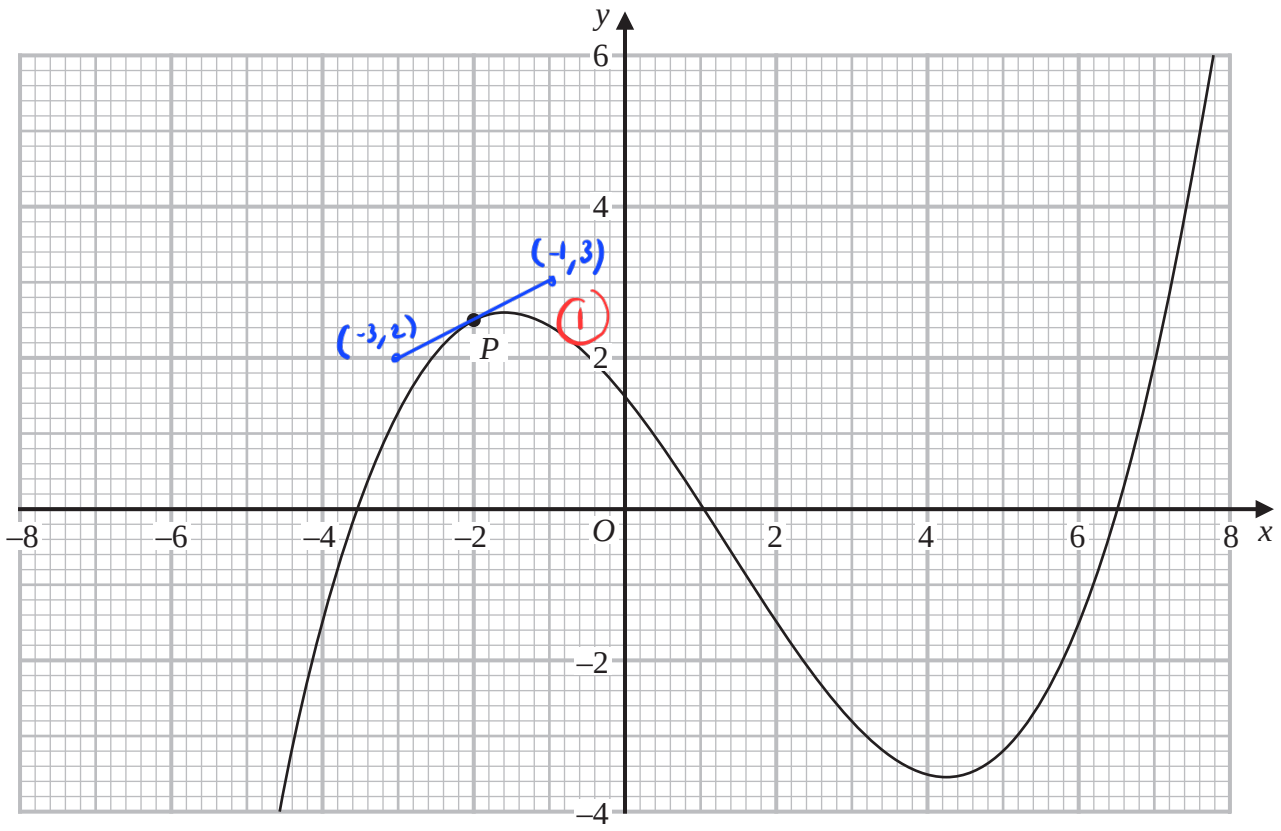
$= 108 - 45 - 36 + 6$

$= 33$ (1)

$y =$ 33

(Total for Question 5 is 6 marks)

6 The diagram shows the graph of $y = f(x)$



The point P has x coordinate -2

Use the graph to find an estimate for the gradient of the curve at P

$m_P = \frac{3-2}{-1-(-3)}$ (1)

$= \frac{1}{2} = 0.5$ (1)

0.5

(Total for Question 6 is 3 marks)

7 The curve **T** has equation $y = x^3 - 2x^2 - 9x + 15$

(a) Find $\frac{dy}{dx}$

$\frac{dy}{dx} = 3x^2 - 4x - 9$ (1)

$\frac{dy}{dx} =$ $3x^2 - 4x - 9$ (1)

(2)

(b) Find the range of values of x for which **T** has a positive gradient.
Give your values correct to 3 significant figures.
Show your working clearly.

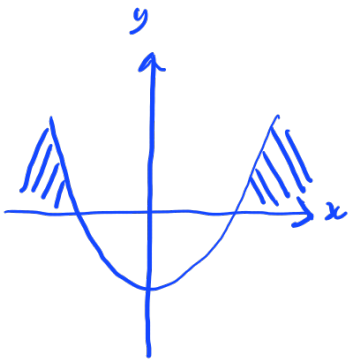
$x = \frac{4 \pm \sqrt{(-4)^2 - 4(3)(-9)}}{6}$ (1)

$= \frac{4 \pm \sqrt{16 + 108}}{6}$

$= \frac{4 \pm \sqrt{124}}{6}$

$= \frac{4 + \sqrt{124}}{6}$ or $\frac{4 - \sqrt{124}}{6}$

$= 2.52\ldots$ or $-1.19\ldots$ (1)



(1) (1)

$x < -1.19$, $x > 2.52$

(4)

(Total for Question 7 is 6 marks)

8 The curve **C** has equation $y = 4x^3 + x^2 - 20x$

(a) Find $\frac{dy}{dx}$

$\frac{dy}{dx} = 12x^2 + 2x - 20$ (2)

$\frac{dy}{dx} =$ $12x^2 + 2x - 20$

(2)

(Total for Question 8 is 2 marks)